Compute the following limits:

(a) $\lim\limits_{x \to 1} \dfrac{x^a - 1}{x^b - 1}$

(b) $\lim\limits_{x \to 0} \dfrac{\sin 5x}{x}$

(c) $\lim\limits_{x \to 0} \dfrac{x^2 - 6x + 2}{x + 1}$

(d) $\lim\limits_{x \to \infty} \dfrac{\ln(1 + e^{3x})}{2x + 5}$

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(a) $\lim_{x \to 1} \dfrac{x^a - 1}{x^b - 1}$

$= \lim_{x \to 1} \dfrac{\dfrac{x^a - 1}{x - 1}}{\dfrac{x^b - 1}{x - 1}}$

$= \dfrac{f'(1)}{g'(1)}$

$= \dfrac{a(1)^{a-1}}{b(1)^{b-1}}$

$= \dfrac{a}{b}$

(b) $\lim_{x \to 0} \dfrac{\sin 5x}{x}$

$f(0) = g(0) = 0$

$\therefore \lim_{x \to 0} \dfrac{\sin 5x}{x}$

$= \dfrac{5\cos 5(0)}{1}$

$= 5$

(c) $\lim_{x \to 0} \dfrac{x^2 - 6x + 2}{x + 1}$

$= \dfrac{0 - 0 + 2}{0 + 1}$

$= 2$

(d) $\lim_{x \to \infty} \dfrac{\ln(1 + e^{3x})}{2x + 5}$

$f(\infty) = g(\infty) = \infty$

and $\lim_{x \to \infty} \dfrac{f'(\infty)}{g'(\infty)}$ exists.

$\therefore \lim_{x \to \infty} \dfrac{\ln(1 + e^{3x})}{2x + 5}$

$= \lim_{x \to \infty} \dfrac{\dfrac{1}{1 + e^{3x}} \cdot 3e^{3x}}{2}$ (L'hospital)

$= \lim_{x \to \infty} \dfrac{1}{2} \cdot \dfrac{9\cancel{e^{3x}}}{3\cancel{e^{3x}}}$ (L'hospital)

$= \dfrac{3}{2}$